

Monetary policy uncertainty and long-term interest rates: A macro-finance approach*

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Abstract

This paper examines the relationship between monetary policy uncertainty and the term structure of interest rates. We present empirical evidence that long-term interest rates are positively related to monetary policy uncertainty, with the magnitude increasing with maturity. The empirical findings suggest that a reduction in measured monetary policy uncertainty would result in a bull-flattening of yield curves. Further, we demonstrate that an affine-term structure model combined with macro-variables and time-varying volatility of monetary policy rule can provide a possible/consistent interpretation behind the empirical facts to conclude that variation of monetary policy uncertainty plays a significant role as a determinant of long-term interest rates.

Key Words: Monetary policy, term structure of interest rates, GARCH.

JEL Classification: E43, E5

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1 Introduction

The aim of our study is to explore the role of uncertainty regarding monetary policy as a determinant of the long term interest rates or yield curves. In this paper, we define the monetary policy uncertainty as the conditional variance of innovations to a monetary policy reaction function. The main analysis in this study is to show how this measured monetary policy uncertainty affects long-term interest rates and yield curves. We first present empirical facts, simple OLS estimation results, which indicate that uncertainty about monetary policy is likely to positively influence long-term interest rates and that the magnitude increases with maturity. In other words, empirical facts suggest that a yield curve becomes bull flattened in response to a reduction in monetary policy uncertainty.

Recently, in the rapidly growing literature of macro-finance, many attempts have been made to examine how macro-variables influence term structure of interest rates. Our study addresses the similar issue paying much attention on the second moment implication, while many earlier studies focus on how changes in (the levels of) macro-state variables affect term premium. Technically, our term structure model including empirical analysis is designed to examine the relationship between yields and (time-varying) volatility stemming from monetary policy. In searching an interpretation of the empirical facts that we found, we employ a version of affine-term structure models. We show that our affine-term structure model which is a variant of that of Bernanke et al (2005) can propose a possible economic interpretation between monetary policy uncertainty and term structure of interest rates.

Among pioneering works in the literature, the affine term structure model introduced in Duffie and Kan (1996) and Dai and Singleton (2002) is a well established approach. These theoretical, but also practically useful models are widely used in finance literature. From viewpoints of macro-economists, a potential flaw of those models might be that they provide little information or interpretation in terms of macroeconomic dynamics, since they depend on latent variables. In those models, generally, we can only guess how the “slope factor” or “level factor” is related to the macroeconomic variables/fluctuations. For instance, in a number of studies one of those latent variables is

likely to be interpreted as unobservable target rate of inflation rate or expected rate of inflation by market participants.¹

In contrast to the “pure finance” models with prevalent convention of relying those latent variables in the literature, including a recent work by Kim and Wright (2005), another string of works such as Ang and Piazzesi (2003), Bernanke et al (2005), Ruedebusch and Wu (2004, 2006), Hoodahl et al (2005) and others combines the affine-term structure model with certain dynamics of macro-variables explicitly, which is now broadly called macro-finance models². Our approach is one of these joint macro-finance models similar to that of Bernanke, Reinhart and Sack (2005), denoted as BRS hereafter. Namely, a combination of a reduced form VAR representation of macro-dynamics and a discrete-time version of affine term structure model á la Duffie and Kan (1996). A new feature of our study is that we explicitly incorporate monetary policy uncertainty, defined as time-varying conditional variance of innovations to a monetary policy rule, into the framework. More specifically, in our model, the measured monetary policy uncertainty derived from GARCH estimation is employed as a component of the state variable vector both in the macro-dynamics and the affine form of yield equations. An appealing feature of our approach is that it can provide more coherent and interpretable results in understanding the variation of risk premium. In a nutshell, this paper is an attempt to explain “unexplained” portion of risk premium focusing the role of perceived monetary policy uncertainty.³

In terms of empirical analysis on monetary policy uncertainty, it is worth mentioning Favero and Mosca (2001) from which our basic idea is developed. These authors presented evidence that monetary policy uncertainty significantly declined around 1994 when the FOMC began to release its target level for the federal funds rate. In this paper, our measurement of monetary policy uncertainty basically follows the specifica-

¹See Ruedebusch and Wu (2004, 2006) for examples.

²In the literature of macro-finance analysis, Ruedebusch and Wu (2004, 2006) style approach is built on structural macroeconomic models, while Ang and Piazzesi (2003) and some others employ reduced form dynamics of macro-variables. Our model can be classified as one in the second group.

³Elder (2001) is based on a motivation related to ours. Namely, Elder (2001) mainly focuses on changes in volatility in inflation, while ours is volatility of innovations to a monetary policy rule.

tion of Favero and Mosca (2001). In the empirical section, we extended their estimation technique to find that the measured monetary policy uncertainty significantly reduced around the period from 2003 to 2005, when the FOMC made an explicit commitment to continuing a low interest rate policy.

Finally we refer to another practical issue related to our study, the “conundrum” dubbed by Greenspan (2005). In his testimony to the US Senate, the Federal Reserve Bank’s (FRB) ex-Chairman Alan Greenspan stated that the “unanticipated behavior of the world bond market remains a conundrum,” indicating the historically low interest rates with long maturities. A number of factors can affect the “low” long-term interest rates.⁴ As for this issue, several works with similar macro-finance approach have already been made and provided various possible explanations.⁵ Although our study does not intend to provide a definitive answer to the conundrum, maybe, our asset pricing model and empirical analysis offer an alternative or additional explanation, which is that the reduction in monetary policy uncertainty could be a potential factor inducing the reduction in long-term interest rates and flattened yield curves during the period between 2003-2005.

In the following section, we present empirical analysis to show that the measured monetary policy uncertainty positively related to long-term interest rates with the magnitude increasing with maturity. Section 3 introduces a no-arbitrage bond price model with time-varying volatility, which is applicable to the findings in the previous section. We discuss the numerical results derived from the model in section 3 in comparison with the empirical facts. Section 5 briefly concludes the paper.

⁴For instance, Greenspan (2005) referred to technical factors (*e.g.*, the behavior of mortgage investors) as a possible explanation of the low yields. However, he commented that none of these factors provides a decisive answer to the conundrum.

⁵See Ruedebusch et al (2006) and Kim and Wright (2005).

2 Empirical analysis

2.1 Measuring monetary policy uncertainty

First, we estimate a standard Taylor-type monetary policy rule to obtain forecast errors as a proxy of monetary policy uncertainty. The specification of the policy rule follows Favero and Mosca (2001). As our estimation frequency is monthly, we cannot use the output gap for an explanatory variable in the policy reaction function since it is available only on a quarterly basis. Instead of the output gap, we use monthly (percent) changes in payroll employment in the non-farm business sector. For inflation and short-term interest rates, we use year-over-year CPI growth rates and euro dollar three-month rates⁶. The estimation period that we apply here is from January 1985 to February 2006. The estimation period is almost the same as that of Ruedebusch et al (2006), where they argue that the observed statistical relationship between macro-variables and interest rates are relatively stable for the period, which is in general considered to be necessary to avoid the well known issue of the Lucas critique. We agree to their view that basically focusing on the “Greenspan era” is less problematic.⁷

Here we take a small detour to have a closer look at the “conundrum.” In addition to the original specification of Favero and Mosca (2001), we add a dummy variable *dum1* in the variance equation as an attempt to capture the possible effect of a commitment announced by the FED in August 2003. The FOMC statement, dated August 12 2003, states, “policy accommodation can be maintained for a considerable period.” FED continued announcement policy via statements with various kind of expressions, which refer to their own future policy directions. At the FOMC in December 2005, they finally removed the kind of commitment phrases from the statement, such as “measured pace” which had been found until that time.

A potentially interesting question to ask here is: did the commitment affect financial

⁶Our data set is almost the same as that of Favero and Mosca (2001). Payroll employment and CPI data are available from the Bureau of Labor Statistics and euro dollar three-month rates are downloadable via the Bloomberg archive. All the data files are available from the authors upon request.

⁷Ruedebusch et al (2006) refer to a number of empirical studies that report some sort of structural breaks around early 1980s to conclude not to include the Volker disinflation era in the estimation period.

markets by reducing uncertainty on monetary policy? The variable *dum1* comprised ones for the period from August 2003 to the end of 2005 and otherwise zeros. As a result, the coefficient on *dum1* would be estimated as significantly negative if the FED's commitment reduced conditional volatility. The estimation result is shown below in Table 1.

[Table 1 here]

As reported in Favero and Mosca (2001), all the coefficients, except for the intercepts, q_0 , in the monetary policy rule are estimated to be positive and statistically significant. Further, the estimation result (spec 2) indicates that the null hypothesis of b_3 being equal to zero is rejected. The result implies that the FED's commitment had a non-negligible effect in reduction of long-term interest rates. We will discuss this issue in section 4 later.

Figure 1 shows the conditional variance based on the GARCH estimation (spec 2) from Table 1. We regard the data in Figure 1 as the measured monetary policy uncertainty. Obviously, as now it is observable data, we can incorporate this variable explicitly both into simple OLS estimations and the no-arbitrage term structure model introduced in the next section.

[Figure 1 here]

Before proceeding to the next table showing simple OLS estimation results, we introduce an alternative measure of monetary policy uncertainty to assess the validity or robustness of the data in Figure 1. In Figure 2, we plot the data of (squared) "unanticipated policy changes" calculated using the methodology suggested by Kuttner (2001).

The data in Figure 2 is calculated using the federal funds future rates, which reflect the average forecast of market participants. Roughly speaking, the difference between the federal funds future rate prior to an FOMC meeting and the actual federal funds rate on the very day of the FOMC meeting can be regarded as the forecast error of

market participants. As the data in Figure 2 is discontinuous (it can be calculated for a date when an FOMC meeting was held), it is impossible to calculate the correlation between the two sequences. However, even a casual observation confirms that the two measurements of monetary policy uncertainty in each figure have a strong positive correlation, implying that the conditional variance in Figure 1 captures well the financial markets' perception of monetary policy uncertainty.

[Figure 2 here]

2.2 Monetary policy uncertainty and yield curves: simple OLS results

Second, we regress the conditional variance estimated above on nominal yields with various maturities. The regression is designed to assess how the changes in monetary policy uncertainty affects the long yields with various maturities. Estimation results are shown below in Table 2 and 3.

[Table 2 and 3 here]

In the tables, $\tilde{\sigma}_{\zeta,t}^2$, Δl_t and π_t denote the conditional variance derived from the GARCH estimation, the monthly changes in payroll employment in the non-farm sector and CPI year-over-year growth rate. R_t^n and R_t^1 are the nominal yield of an n -year bond and euro dollar three month rate respectively. Later, we will show that the regressors vector $Y_t = \left(R_{t-1}^1 \ \Delta l_t \ \pi_t \ \tilde{\sigma}_{\zeta,t}^2 \right)'$ is exactly the same as the state variable vector in our no-arbitrage bond pricing model. That is, we could consistently interpret the empirical fact indicated by the simple OLS estimation with the formal affine term structure model to be presented later. In contrast to some early studies, we exclude observable expectations variables from regressors, such as survey-based expected inflation rate, TIPS spread or federal funds future rates.⁸ Incorporating these variables into the estimation might give

⁸See Ruedebusch et al (2006) for example.

us a better fit of the model, but it comes at the cost of a certain inconsistency in the term structure model. We will discuss this issue later in section 3.

As shown in the tables, all the coefficients are positive and statistically significant. The outstanding result is that the coefficients on monetary policy uncertainty tend to be larger as the maturity increases. This fact immediately implies that a reduction in monetary policy uncertainty represented by $\tilde{\sigma}_{\zeta,t}^2$ induces bull-flattening of yield curves. Considering the lower $\tilde{\sigma}_{\zeta,t}^2$ during the period from 2003 to 2005, some reduction in monetary policy uncertainty in this period is likely to have played some roles in creating the conundrum in treasury markets.

One might argue that uncertainty about long-run target level of inflation rate of the FED creates larger sensitivity in long-term interest rates.⁹ We do not exclude such possibility per se, but it should be noted that the focus here is the second moment of the error term, that is, $\tilde{\sigma}_{\zeta,t}^2$. Naturally, deviations in any direction, either upward or downward, from predicted values by the estimated monetary policy rule seems to uniformly positively affect the longer-term yields. This is in sharp contrast to the “uncertain inflation target hypothesis,” which predicts that an upward error in the federal funds target lowers the long-term nominal yields due to the downward revision of perceived long-run target level of inflation by market participants.¹⁰ (And vice versa in case of downward errors.) Since the square terms $\tilde{\sigma}_{\zeta,t}^2$ do matter here as shown in table 2 and 3, this cannot be attributed to changes in a perceived *level* of target rate of inflation.

Another observation is, though less important, the coefficients on Δl_t tend to become smaller as the maturity increases. We will discuss the theoretical foundation behind these empirical findings in the following section.

3 A macro-finance model of term structure

In this section, we discuss possible interpretation of the empirical findings presented in the previous section. For the purpose, we introduce a no-arbitrage bond pricing model

⁹See Gürkaynak et al (2005) for the argument.

¹⁰This phenomenon is known as “excess sensitivity” of longer term forward rates. See Gürkaynak et al (2005).

combined with a reduced form macroeconomic dynamics represented by a simple VAR. Before proceeding to the model presentation, here is one caveat. In this paper, we are not trying to explain all the observed variations of yield curves by the simple model, but our aim is to demonstrate that a relatively simple model, which is consistent with the specification of the OLS estimation presented in section 2 could provide a theoretical underpinning *qualitatively*. To obtain a good match of the model with actual data in every aspect, the model must contain broad range of stochastic disturbances other than monetary policy uncertainty. This comes at the cost of loss of clarity in showing the role of monetary policy uncertainty which we focus on in this paper. Hence, although we have carefully chosen sufficient numbers of the state variable incorporated into the model, our discussion basically concentrate on qualitative aspect of the issue, keeping the model minimum.

3.1 Macroeconomic dynamics: a VAR representation

Consider a simple four-variable VAR(1). The state variable vector Y_t consists of π_t , Δl_t , R_t^1 and $\sigma_{\zeta,t}^2$, each of which denote CPI inflation, changes in payroll employment, euro-dollar three month rates and (time-varying) monetary policy uncertainty. There are essentially two reasons in choosing these variables as the state variables in the model. First, by construction of the affine term structure model presented later, market interest rates with any maturities should generally share the state variables, although some of the parameter can be zeros. As we employ the standard Taylor rule, depending lagged short-term rates, CPI inflation rate and payroll employment, it is a natural consequence that longer term yields share these state variables as their determinants. Another reason for choosing observed/actual variables rather than expectations variables, such as the Blue Chip based expected inflation or TIPS spread is to avoid internal inconsistency in the model. We estimate a VAR to represent macroeconomic dynamics, which can provide one or several period ahead forecasted values of the state variables. Not surprisingly, the VAR based forecast and survey-based expectations do not coincide in general. Potentially we could use either or both of them, but here in this paper we employ actual data (*e.g.* realized/observed CPI data) to put much importance on internal consistency.

Now the four-variable VAR can be written as,

$$\begin{aligned} \begin{bmatrix} R_t^1 \\ \Delta l_{t+1} \\ \pi_{t+1} \\ \sigma_{\zeta,t+1}^2 \end{bmatrix} &= \begin{bmatrix} \psi & q_1 & q_2 & q_3 \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ 0 & 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} R_{t-1}^1 \\ \Delta l_t \\ \pi_t \\ \sigma_{\zeta,t}^2 \end{bmatrix} + \begin{bmatrix} \zeta_{t+1} \\ \varepsilon_{t+1} \\ v_{t+1} \\ e_{t+1} \end{bmatrix} \\ &\equiv \mathbf{F}Y_t + \mathbf{e}_{t+1} \end{aligned} \quad (1)$$

where each element in $(\varepsilon_t \ v_t \ e_t)'$ distributes normal with the variance $(\sigma_\varepsilon^2 \ \sigma_v^2 \ \sigma_e^2)$. As we discussed in section 2, we allow heteroskedasticity in ζ_t . For the estimation of the first row of the \mathbf{F} matrix, we just embed the GARCH estimation result (spec 1) in table 1. In doing so, the expected inflation term $E_t\pi_{t+1}$ can be substituted out by,

$$E_t\pi_{t+1} = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{F}Y_t. \quad (2)$$

Essentially, for eqn (2), only the third row in the \mathbf{F} matrix matters here. Since any VAR estimation can be done independently for each row, the estimated coefficients in the third row are not affected by the GARCH estimation or estimation of the first row of the \mathbf{F} . That is, we can safely estimate the first row of \mathbf{F} separately from the rest of them. Since we have already estimated the GARCH model in section 2, we can directly exploit the results here. Plugging the eqn (2) into the GARCH model shown in table 1, we obtain the estimates of parameters in the first row of \mathbf{F} matrix, $F_1 \equiv (\psi \ q_1 \ q_2 \ q_3)$.

$$\begin{aligned} R_t^1 &= \tilde{\psi}R_{t-1}^1 + \tilde{q}_1\Delta l_t + \tilde{q}_2E_t\pi_{t+1} + \zeta_t \\ &= \tilde{\psi}R_{t-1}^1 + \tilde{q}_1\Delta l_t + \tilde{q}_2 \begin{bmatrix} f_{31} & f_{32} & f_{33} & f_{34} \end{bmatrix} Y_t + \zeta_t \\ &= \begin{bmatrix} \psi & q_1 & q_2 & q_3 \end{bmatrix} Y_t + \zeta_t. \end{aligned} \quad (3)$$

Again note that the vector $F_3 \equiv (f_{31} \ f_{32} \ f_{33} \ f_{34})$ can be estimated by OLS separately as a part of the VAR estimation. Similarly, $F_2 \equiv (f_{21} \ f_{22} \ f_{23} \ f_{24})$ can be estimated by OLS.

Now, let us turn to the fourth rows of the \mathbf{F} and Y_t . The fourth element in Y_t represents time-varying conditional variance of ζ_t , as we allow heteroskedasticity in innovations to the monetary policy rule. We employ the time series data of estimated

conditional variance shown in Figure 1. As for the fourth row of the \mathbf{F} , we constrain the parameters as shown in eqn (1), such that the first three entities are constrained to zeros. This constraint implies that dynamics of the time-varying monetary policy uncertainty follows simple ARCH(1) process. In other words, it is not affected by other state variables such as inflation or payroll employment. This constraint might seem somewhat ad-hoc, but it is assumed in the spirit of the GARCH estimation results in section 2. Following the GARCH estimation results, we calibrated the value of ρ to be equal to 0.96, which is the sum of b_1 and d_1 in table 1 (spec1). The estimated VAR parameters are summarized in table 4.

Now we are ready to add a bond pricing formula to this framework to analyze term premium.

3.2 The affine term structure model with time-varying volatility

We consider a standard discrete-time affine term structure model (ATSM) following Duffie and Kan (1996).¹¹ As commonly employed in the literature, we build our model imposing the assumption of no riskless arbitrage opportunities among bonds of various maturities, which assures the existence of a risk neutral measure. The only extension to the standard ATSM is that we allow time-varying volatility with a state transition derived from the VAR as presented above.

First, for the purpose of handling time-varying volatility, it is convenient to re-define ζ_t as,

$$\zeta_t = \sqrt{\sigma_t^2} \times \tilde{\zeta}_t,$$

where $\tilde{\zeta}_t \sim N(0, 1)$. For a stochastic process of $\sigma_{\zeta,t}^2$, as we have discussed, simple ARCH(1) process is assumed such that,

$$\sigma_{\zeta,t}^2 = \rho\sigma_{\zeta,t-1}^2 + \sqrt{\sigma^2} \times \tilde{e}_t, \quad \rho < 1, \quad (4)$$

where $\tilde{e}_t \sim N(0, 1)$. Similarly, other stochastic disturbances ($\tilde{\varepsilon}_t$ \tilde{v}_t) are redefined accordingly; $\varepsilon_t = \sqrt{\sigma_\varepsilon^2} \times \tilde{\varepsilon}_t$ and $v_t = \sqrt{\sigma_v^2} \times \tilde{v}_t$. Applying the redefined stochastic disturbances,

¹¹Our model provided here is nearly identical to a discrete-time analogs of Duffie and Kan (1996) that is developed by Rudebusch and Wu (2004, 2006).

where p_t^1 denotes the price of one-period bond at period t . Note that $A_1 (= \delta_0)$ is a scalar and $B_1 (= \delta_1)$ is a 1×4 parameter vector. To obtain a bond pricing formula, we need to specify how the price of risk is determined. We slightly modified the assumption of Duffee (2002) and others,¹² so that the “sigma-adjusted” price of risk ($= \Sigma_t \Omega_t$) assumed to be affine in the state variables. Namely,

$$\Sigma_t \Omega_t = \Omega_0 + \Omega_1 Y_t, \quad (6)$$

where Ω_0 is a 4×1 parameter vector and Ω_1 is a 4×4 parameter matrix, both of which are to be estimated. As for this “quasi-affine form” assumption for the risk price vector, we discuss more in details in appendix.

It can be shown that the above structure of the model implies that the n -period log bond price is also affine in Y_t . That is,

$$\ln(P_t^n) = -A_n - B_n Y_t,$$

where A_n is a scalar and B_n is a 1×4 vector.

With the structure presented above, no arbitrage condition creates the evolution of A_n and B_n as shown below.

$$\begin{aligned} A_n &= A_{n-1} - B_{n-1} \Omega_0 - \frac{1}{2} B_{n-1} \Sigma_c \Sigma_c' B_{n-1}' + \delta_0 \\ B_n &= B_{n-1} (\mathbf{F} - \Omega_1) - \frac{1}{2} (B_{n-1}(1))^2 s_4 + \delta_1, \end{aligned}$$

where $B_n(1)$ denotes the first element in B_n and

$$\Sigma_c = \begin{bmatrix} 0 & & & \\ \sigma_{\zeta\varepsilon} & \sigma_\varepsilon^2 & & \\ \sigma_{\zeta v} & \sigma_{\varepsilon v} & \sigma_v^2 & \\ 0 & 0 & 0 & \sigma_e^2 \end{bmatrix}.$$

¹²In the standard C-CAPM literature, the price of risk is related to the shape of investors/households’ utility function, rather than postulated exogenously. There are, however, a number of studies that assume more flexible affine form similar to ours. See, Dai and Singleton (2002), Hördahl et al (2004), Rudebusch and Wu (2004), Kim and Wright (2005) and others. Among them, Hördahl et al (2004) argue relative advantages of this formulation in details.

Detailed derivation of the evolution of A_n and B_n is discussed in appendix. The upshot of the above argument is that yields of n -period bonds R_t^n are expressed as,

$$R_t^n = -\frac{1}{n} \ln(p_t^n) \quad (7)$$

$$= \frac{1}{n} A_n + \frac{1}{n} B_n Y_t \equiv \bar{A}_n + \bar{B}_n Y_t. \quad (8)$$

The eqn (7) is comparable with the OLS estimation results in section 2 due to the notable feature that \bar{A}_n and \bar{B}_n are independent of time-varying monetary policy uncertainty $\sigma_{\zeta,t}^2$ in Σ_t . Further, this equation (7) allows us to examine how the entire shape of yield curves responds to changes in monetary policy uncertainty $\sigma_{\zeta,t}^2$. The ATSM introduced here, with the estimated market price of risk in eqn (6), confirms the prediction by the OLS estimations results, which we discuss in details in next section.

4 The ATSM estimation results

4.1 The ATSM results

As discussed so far, we conducted two-stage estimation for the VAR dynamics and factor loadings in the market price of risk following BRS or Ruedebusch et al (2006). The risk price vector is estimated by non-linear least squares (NLS), since both A_n and B_n are highly non-linear functions in Ω_0 and Ω_1 . We started the estimation with an initial values by calibration, which gives a seemingly good fit for 10 year yield for the sample period, from 1986 to 2006. Since there are 14 ($= 4 + 4 \times 4 - 6$) free parameters in eqn (6), there might be multiple local minima in the NLS estimation. We focus on the estimates that minimizes the sum of squared residuals from 10 year yield estimation. As we stated, we are not trying to explain all the variations of entire yields curves at once, but rather focusing on the relationship between long-term interest rates, especially 10 year yield, and monetary policy uncertainty. The estimated factor loadings in the risk of price equation are presented in table 5.¹³

Figure 3 shows predicted values of 10 year yield by our ATSM in comparison with

¹³Matlab codes for the computation of yield cuves are available at the author's web site: www.ryokato.org

the actual data.¹⁴ Although our model does not depend on any latent variables and employs only a small number of, — actually only four — state variables, the predicted values show a good fit even quantitatively. Nonetheless, we observe relatively large deviation between the predicted value and actual data around 2005, which can be not yet explained portion of the conundrum. We will discuss this issue later in next section.

[Figure 3 here]

Figure 4 depicts the evolution of \bar{B}_n of the ATSM. A notable feature of our ATSM is that the fourth element in \bar{B}_n , that is, the coefficient on $\tilde{\sigma}_t^2$, has a smooth upward trend as the maturity increases. In other words, $\partial^2 R_t^n / \partial n \partial \sigma_t^2 > 0$. Obviously this is consistent with the OLS estimation results presented in section 2. Namely,

$$\left(\frac{\partial R_t^{120}}{\partial \sigma_t^2} \right)_{OLS} > \left(\frac{\partial R_t^{60}}{\partial \sigma_t^2} \right)_{OLS} > \left(\frac{\partial R_t^{12}}{\partial \sigma_t^2} \right)_{OLS},$$

both of which imply bull-flattening of a yield curve in response to a reduction to monetary policy uncertainty.

[Figure 4 here]

The comparison of the ATSM coefficients with those of the OLS results are shown below:

$$\begin{aligned} \bar{B}_{120}^{ATSM} &= \begin{bmatrix} 0.576 & 1.543 & 0.200 & 6.942 \end{bmatrix}, \\ \bar{B}_{120}^{OLS} &= \begin{bmatrix} 0.424 & 0.866 & 0.243 & 9.977 \end{bmatrix}, \end{aligned}$$

where \bar{B}_{120}^{ATSM} and \bar{B}_{120}^{OLS} denote coefficient vector in 10 year yield equation; $R_t^{120} = A_{120} + B_{120}Y_t$ for each model. The two B_{120} vectors shown above seem fairly close, a part of which reason is that we estimated the factor loadings in the ATSM so that they

¹⁴Rigrouly speaking, the predicted values by the ATSM are yields of zero coupon bonds, which are different from the compared actual data shown in figure 3. But note that our focus in the empirical section is the most prevalent 10 year yield data from the FRB statistics, which is displayed in figure 3.

give the best fit for the 10 year yield data. Admittedly, a shorter-term yield, such as B_{24}^{ATSM} vector is less close to that of the OLS, and the fit is not that good either. But note that the OLS estimators are not subject to the no-arbitrage condition as opposed to the ATSM.

$$\begin{aligned}\bar{B}_{24}^{ATSM} &= \begin{bmatrix} 0.022 & 0.012 & 0.070 & 0.737 \end{bmatrix}, \\ \bar{B}_{24}^{OLS} &= \begin{bmatrix} 0.770 & 1.530 & 0.12 & 2.29 \end{bmatrix}.\end{aligned}$$

4.2 Discussion: an answer to the conundrum?

Since the fourth element in \bar{B}_{120}^{ATSM} is positive, a reduction in monetary policy uncertainty would induce a lower 10 year yield. As we calculated based on the ATSM parameter and the time series data in figure 1 (spec 1; without the dummy variable), reduction in monetary policy uncertainty explains 33.8 percent of the fall of US 10 year yield from 2002 to the end of 2005. The same calculation is possible by the OLS coefficients. In this case, a larger portion, 48.4 percent of the fall can be attributed to the reduction in monetary policy uncertainty for the same period.

As appears in Figure 3, the predicted values of the both two models deviate upward from the actual data around 2004-2005. The best fit is obtained from the OLS estimates combined with the spec 2 data of the GARCH result that includes dummy variable. The dummy variable captures possible influence of the commitment or announcement by the FED during the period from August 2003 to the end of 2005. In this sense, it is likely that, more or less, relatively more forecastable monetary policy during the period played a certain role in reducing the long-term yield or risk premium.¹⁵ But by carefully looking at Figure 3, we realize that the reduction in monetary policy uncertainty or more forecastable monetary policy started to prevail sometime around 2002. This fact may imply that the ultimate source of the more forecastable monetary policy is not the commitment per se, but broader range of maneuvers of the Fed associated with more

¹⁵Ruedebusch et al (2006) argue that reduction in the implied volatility of long-term interest rates have a significant power in explaining the lower interest rates for the same period. Our view might enhance theirs in the sense that some reduction of monetary policy uncertainty can be an ultimate source behind such lower implied volatility of longer-term yields.

stable economic fluctuations in the period.

[Figure 5 here]

Having said that, it is fairly evident that our ATSM and even the simple OLS estimates cannot explain the entire variation of the long-term yields. A straightforward extension to cope with the remaining problem is to include more state variables, such as lagged values of our Y_t or completely new ones. In this paper, we do not pursue further in that direction, since as we stated repeatedly, our focus is to analyze the role of $\sigma_{\zeta,t}^2$ in a simple and tractable ATSM. Incorporating the monetary policy uncertainty represented by $\sigma_{\zeta,t}^2$ into macro-finance framework is just a first step to capture the real/unobservable uncertainty, and therefore, there is a further way to go in this direction.

5 Concluding remarks

This paper presents the empirical facts that the long-term interest rates are positively related to monetary policy uncertainty with the magnitude increasing with maturity. Further, we demonstrate that a simple macro-finance model could mimic the substantive portion of the empirical fact that we found. An advantage of our approach is that our model does not rely on latent variables, but solely consists of observable economic variables. Due to the feature of the model, we could explicitly interpret the variations of the long-term interest rates in accordance with the no-arbitrage condition.

In spite of its simplicity and superior performance, a number of possible concerns can be raised as for the ATSM that we propose here in this paper. Among them, the assumption of rational expectations formulated by market participants combined with time-invariant target rate of inflation of the FED might seem unrealistic. Forecast errors obtained in the GARCH estimation can be attributable to time-varying target rate of inflation or fluctuating perceived target rate of inflation, both of which are inherently unobservable. We do not pursue further to pin down the fundamental source of monetary policy uncertainty in this paper, but the question remains open to future research.

References

- [1] Ang, A. and M. Piazzesi, 2003, A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, *Journal of Monetary Economics* 50, pp. 745-787.
- [2] Bernanke, B., V. Reinhart and B. Sack, 2005, Monetary policy alternatives at the zero bound: An empirical assessment, *Brookings Papers on Economic Activity* 2, 1-78.
- [3] Duffie, D. and R. Kan, 1996, A yield factor model of interest rates, *Mathematical Finance* 6, pp. 379-406.
- [4] Duffee, G. R. 2002, Term premia and interest rate forecasts in affine models, *Journal of Finance* 57, pp. 405-443.
- [5] Elder, J., 2001, Can the volatility of the federal funds rate explain the time-varying risk premium in treasury bill returns? *Journal of Macroeconomics* 23, pp. 73-97.
- [6] Engel, R. F. and T. Bollerslev, 1986, Modeling the persistence of conditional variances, *Econometric Reviews* 5, pp. 1-50.
- [7] Favero, C. A. and F. Mosca, 2001, Uncertainty on monetary policy and the expectation model of the term structure of interest rates, *Economics Letters* 71, pp. 369-375.
- [8] Greenspan, A., 2005, Federal Reserve Board's semiannual monetary policy report to the Congress, Board of Governors.
- [9] Gurkaynak, R., B. Sack and E. Swansson 2005, The sensitivity of long-term interest rates to economic news: Evidence and implications for macroeconomic models, *American Economic Review* 95, pp. 425-436.

- [10] Hoordahl, P, O. Tristani and D. Vestin, 2004, A joint econometric model of macro-economic and term structure dynamics, *Journal of Econometrics* 131, 405-444.
- [11] Kim, D. and J. H. Wright, 2005, An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates, Board of Governors Finance and Economics Discussion Paper, 2005-33.
- [12] Kuttner, K. N., 2001, Monetary policy surprises and interest rates: Evidence from the Fed funds futures market, *Journal of Monetary Economics* 47, pp. 523-544.
- [13] Ruedebusch, G. D. and T. Wu, 2004, A macro-finance model of the term structure, monetary policy, and the economy, Federal Reserve Bank of San Francisco Proceedings, March 2004.
- [14] Ruedebusch, G. D. and T. Wu, 2006, Accounting for a shift in term structure behavior with no-arbitrage and macro-finance models, forthcoming in *Journal of Money, Credit and Banking*.

A Appendix: ATSM with time-varying volatility

A.1 Quasi-affine form of the market price of risk

Our formulation of the market price of risk is basically taken from Duffee (2002). In our paper, the “quasi-affine form” of the market price of risk ($= \Omega_t$) is assumed, which can be denoted as,

$$\Omega_t = \check{\Sigma}_t \Omega_0 + \check{\Sigma}_t \Omega_1 Y_t,$$

where $\check{\Sigma}_t$ stands for a certain symmetric time-variant matrix, which is assumed to be full-ranked. By the assumption, $\Sigma_t^{-1} \equiv \check{\Sigma}_t$ can be defined. Thus, the formulation is now rewritten as,

$$\Omega_t = \Sigma_t^{-1} \Omega_0 + \Sigma_t^{-1} \Omega_1 Y_t.$$

Or simply,

$$\Sigma_t \Omega_t = \Omega_0 + \Omega_1 Y_t,$$

which indicates our formulation, such that the “sigma-adjusted” market price of risk is affine in the state variables.

A.2 The ATSM with time-varying monetary policy uncertainty

The argument here is an extension of Ruedebusch and Wu (2004) with the proof of the existence of the affine form given in Duffie and Kan (1996).

Suppose that the log of a n -period bond price is affine in the state variable vector as discussed in Duffie and Kan (1996).

$$\begin{aligned} \ln(p_t^n) &= -A_n - B_n Y_t \\ p_t^n &= \exp(-A_n - B_n Y_t). \end{aligned}$$

And the state transition as shown in eqn (5),

$$Y_t = \mathbf{F} Y_{t-1} + \Sigma_t \tilde{\mathbf{e}}_t,$$

of which alternative expression under another martingale major is defined as,

$$Y_t = \alpha + \mathbf{F}^Q Y_{t-1} + \Sigma_t \tilde{\mathbf{e}}_t^Q.$$

With the assumption $\Sigma_t \Omega_t = \Omega_0 + \Omega_1 Y_t$. This can be rewritten as

$$\begin{aligned} Y_t &= \alpha + \mathbf{F}^Q Y_{t-1} + \Sigma_t \Omega_t + \Sigma_t \tilde{\mathbf{e}}_t \\ &= (\alpha + \Omega_0) + (\mathbf{F}^Q + \Omega_1) Y_{t-1} + \Sigma_t \tilde{\mathbf{e}}_t \\ &= \mathbf{F} Y_{t-1} + \Sigma_t \tilde{\mathbf{e}}_t. \end{aligned}$$

Hence we have, $\alpha + \Omega_0 = 0$ and $\mathbf{F}^Q + \Omega_1 = \mathbf{F}$.

Now, no-arbitrage condition requires,

$$\begin{aligned} E_t^Q \left(\frac{p_{t+1}^{n-1}}{p_t^n} \right) &= E_t^Q \left[\frac{\exp(-A_{n-1} - B_{n-1} Y_{t+1})}{\exp(-A_n - B_n Y_t)} \right] \\ &= E_t^Q \exp(-A_{n-1} - B_{n-1} Y_{t+1} + A_n + B_n Y_t) \\ &= \exp \left(A_n - A_{n-1} - B_{n-1} (\alpha + \mathbf{F}^Q Y_t) + B_n Y_t + \frac{1}{2} B_{n-1} \Sigma_t \Sigma_t' B_{n-1}' \right) \textcircled{9} \\ &= \frac{1}{p_t^n} = \exp(-A_1 - B_1 Y_t). \end{aligned}$$

By construction of the (G)ARCH model, $\Sigma_t \Sigma_t'$ can be decomposed into two parts as follows.

$$\begin{aligned} \Sigma_t \Sigma_t' &= \begin{bmatrix} \sigma_{\zeta,t}^2 & & & \\ \sigma_{\zeta\varepsilon} & \sigma_\varepsilon^2 & & \\ \sigma_{\zeta v} & \sigma_{\varepsilon v} & \sigma_v^2 & \\ \sigma_{\zeta e} & \sigma_{\varepsilon e} & \sigma_{ve} & \sigma_e^2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & & & \\ \sigma_{\zeta\varepsilon} & \sigma_\varepsilon^2 & & \\ \sigma_{\zeta v} & \sigma_{\varepsilon v} & \sigma_v^2 & \\ \sigma_{\zeta e} & \sigma_{\varepsilon e} & \sigma_{ve} & \sigma_e^2 \end{bmatrix} + \begin{bmatrix} \sigma_{\zeta,t}^2 & & & \\ 0 & 0 & & \\ 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ &\equiv \Sigma_c \Sigma_c' + \boldsymbol{\sigma}_t^2 \end{aligned}$$

Note that we assume non-diagonal elements in $\Sigma_t \Sigma_t'$ or covariance terms are all time-invariant. In addition, in our case, $(\sigma_{\zeta e} \sigma_{\varepsilon e} \sigma_{ve}) = (0 \ 0 \ 0)$. Hence,

$$\begin{aligned} B_{n-1} \Sigma_t \Sigma_t' B_{n-1}' &= B_{n-1} (\Sigma_c \Sigma_c' + \boldsymbol{\sigma}_t^2) B_{n-1}' \\ &= B_{n-1} \Sigma_c \Sigma_c' B_{n-1}' + (B_{n-1}(1))^2 \sigma_{\zeta,t}^2 \end{aligned}$$

Recall that $\sigma_{\zeta,t}^2 = (0 \ 0 \ 0 \ 1) Y_t$. Let the selecting vector $s_4 = (0 \ 0 \ 0 \ 1)$. Then,

$$B_{n-1} \Sigma_t \Sigma_t' B_{n-1}' = B_{n-1} \Sigma_c \Sigma_c' B_{n-1}' + (B_{n-1}(1))^2 s_4 Y_t. \quad (10)$$

where $B_n(1)$ denotes the first element in B_n . Replacing $B_{n-1} \Sigma_t \Sigma_t' B_{n-1}'$ in eqn (1) by eqn (2) yields,

$$E_t^Q \left(\frac{p_{t+1}^{n-1}}{p_t^n} \right) = \exp \left[A_n - A_{n-1} - B_{n-1} Y_{t+1} + B_n Y_t + \frac{1}{2} \left(B_{n-1} \Sigma_c \Sigma_c' B_{n-1}' + (B_{n-1}(1))^2 s_4 Y_t \right) \right].$$

Therefore, we have,

$$\begin{aligned} A_n - A_{n-1} - B_{n-1} \alpha + \frac{1}{2} B_{n-1} \Sigma_c \Sigma_c' B_{n-1}' &= -A_1 \\ -B_{n-1} \mathbf{F}^Q + B_n + \frac{1}{2} (B_{n-1}(1))^2 s_4 &= -B_1. \end{aligned}$$

Plugging the two relations, $\alpha + \Omega_0 = 0$ and $\mathbf{F}^Q + \Omega_1 = \mathbf{F}$ to obtain,

$$\begin{aligned} A_n - A_{n-1} + B_{n-1} \Omega_0 + \frac{1}{2} B_{n-1} \Sigma_c \Sigma_c' B_{n-1}' &= -A_1 \\ -B_{n-1} (\mathbf{F} - \Omega_1) + B_n + \frac{1}{2} (B_{n-1}(1))^2 s_4 &= -B_1. \end{aligned}$$

Rearranging these two to finally yields,

$$\begin{aligned} A_n &= A_{n-1} - B_{n-1} \Omega_0 - \frac{1}{2} B_{n-1} \Sigma_c \Sigma_c' B_{n-1}' + \delta_0 \\ B_n &= B_{n-1} (\mathbf{F} - \Omega_1) - \frac{1}{2} (B_{n-1}(1))^2 s_4 + \delta_1, \end{aligned}$$

where $A_1 = \delta_0$ and $B_1 = \delta_1$.

Table 1: GARCH estimation

<u>Monetary policy rule</u>					
$R_t^1 = \tilde{\psi}R_{t-1}^1 + \tilde{q}_1\Delta l_t + \tilde{q}_2E_t\pi_{t+1} + q_0 + \tilde{\zeta}_t$					
	$\tilde{\psi}$	\tilde{q}_1	\tilde{q}_2	q_0	S.E.
Spec1	0.964 [0.007]	0.720 [0.079]	0.051 [0.013]	-0.076 [0.042]	0.217
Spec2	0.963 [0.006]	0.712 [0.088]	0.053 [0.014]	-0.075 [0.043]	0.219
<u>Variance equation</u>					
$\tilde{\sigma}_{\zeta,t}^2 = b_1\tilde{\zeta}_{t-1}^2 + b_2\tilde{\sigma}_{\zeta,t-1}^2 + b_3 \times dum1 + b_0$					
	b_1	b_2	b_3	b_0	
Spec1	0.129 [0.042]	0.836 [0.047]	-	0.00 [0.00]	
Spec2	0.127 [0.048]	0.819 [0.064]	-0.001 [0.000]	0.00 [0.04]	
White test for heteroskedasticity					
$w=34.32$, p-value=0.00					

Note: Sample period is from Jan. 1986 to Feb. 2006. Numbers in [] are standard errors.

Table 2: OLS estimation with $\tilde{\sigma}_{\zeta,t}^2$ from spec 1

$$R_t^n = \gamma_1 R_{t-1}^1 + \gamma_2 \Delta l_t + \gamma_3 \pi_t + \gamma_4 \tilde{\sigma}_{\zeta,t}^2 + \gamma_0 + \eta_t$$

sample\maturity		1-year	2-year	5-year	10-year
	γ_1	0.84** [0.01]	0.77** [0.02]	0.57** [0.03]	0.42** [0.03]
Jan.1986	γ_2	1.44** [0.39]	1.53** [0.26]	1.08* [0.34]	0.86* [0.39]
-Feb.2006	γ_3	0.10** [0.03]	0.12** [0.04]	0.19** [0.05]	0.24** [0.06]
	γ_4	-0.34 [0.75]	2.29* [1.03]	6.82** [1.38]	9.98** [1.56]
	γ_0	0.21** [0.78]	0.77** [0.11]	2.01** [0.14]	2.87** [0.16]
	S.E.	0.379	0.517	0.694	0.783
	R ² -adj	0.965	0.932	0.838	0.755

Note: Numbers in [] are standard errors. ** and * denote estimates are significant at 1% and 5% level.

Table 3: OLS estimation with $\tilde{\sigma}_{\zeta,t}^2$ from spec 2

$$R_t^n = \gamma_1 R_{t-1}^1 + \gamma_2 \Delta l_t + \gamma_3 \pi_t + \gamma_4 \tilde{\sigma}_{\zeta,t}^2 + \gamma_0 + \eta_t$$

sample\maturity		1-year	2-year	5-year	10-year
	γ_1	0.84** [0.02]	0.767** [0.02]	0.557** [0.03]	0.41** [0.03]
Jan.1986	γ_2	1.44** [0.19]	1.55** [0.25]	1.11** [0.34]	0.1* [0.39]
-Feb.2006	γ_3	0.10** [0.03]	0.126** [0.04]	0.20** [0.05]	0.258** [0.06]
	γ_4	-0.41 [0.83]	2.75* [1.14]	8.24** [1.52]	12.00** [1.70]
	γ_0	0.21** [0.78]	0.760** [0.11]	1.97** [0.14]	2.81** [0.16]
	S.E.	0.379	0.516	0.687	0.771
	R ² -adj	0.965	0.932	0.841	0.762

Note: Numbers in [] are standard errors. ** and * denote estimates are significant at 1% and 5% level.

Table 4: VAR representation

$$Y_t = \mathbf{F}Y_{t-1} + \mathbf{e}_t$$

sample\variable	R_{t-1}^1	Δl_t	π_t	$\tilde{\sigma}_{\zeta,t}^2$	
	R_{t-1}^1	0.969	0.722	0.0495	0.0065
Jan.1986	Δl_t	0.008* [0.004]	0.578** [0.05]	-0.021** [0.008]	0.141 [0.215]
-Feb.2006	π_t	0.005 [0.012]	0.219 [0.149]	0.9577** [0.02]	0.125 [0.60]
	$\tilde{\sigma}_{\zeta,t}^2$	0.0	0.0	0.0	0.96

Note: Numbers in [] are standard errors. ** and * denote estimates are significant at 1% and 5% level.

Table 5: NLS estimation for the market price of risk

		$\Sigma_t \Omega_t = \Omega_0 + \Omega_1 Y_t$			
		ζ	ε	v	e
Ω_1	ζ	1.550			
	ε	0.990	0.280		
	v	-0.010	-0.028	-0.010	
	e	-0.150	-0.065	-0.005	-0.110
Ω_0		0.050	0.020	-0.100	-0.030

Table 6: The ATSM result summary

$$R_t^n = \bar{A}_n + \bar{B}_n Y_t$$

sample\maturity		2-year	3-year	5-year	10-year
	$\bar{B}_n(1)$	0.0218	0.0501	0.0886	0.1064
Jan.86	$\bar{B}_n(2)$	0.117	0.172	0.3692	0.8932
-Feb.2006	$\bar{B}_n(3)$	0.070	0.1169	0.3072	0.8948
	$\bar{B}_n(4)$	0.7273	1.4891	3.2274	6.9326

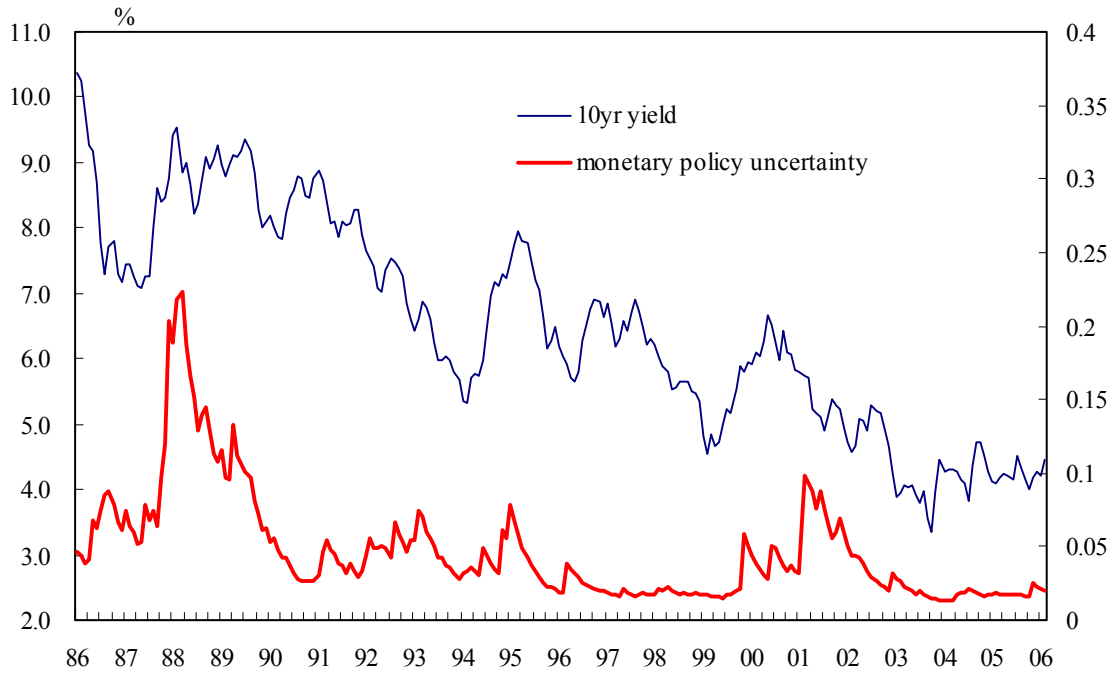


Figure1 : Monetary policy uncertainty

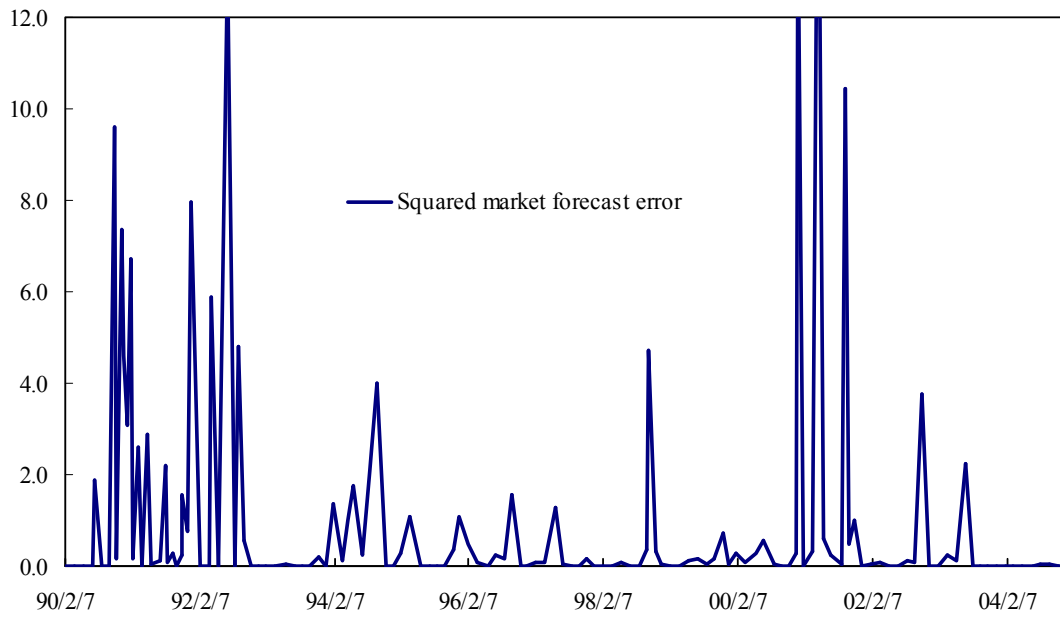


Figure 2: Monetary policy uncertainty—alternative measure

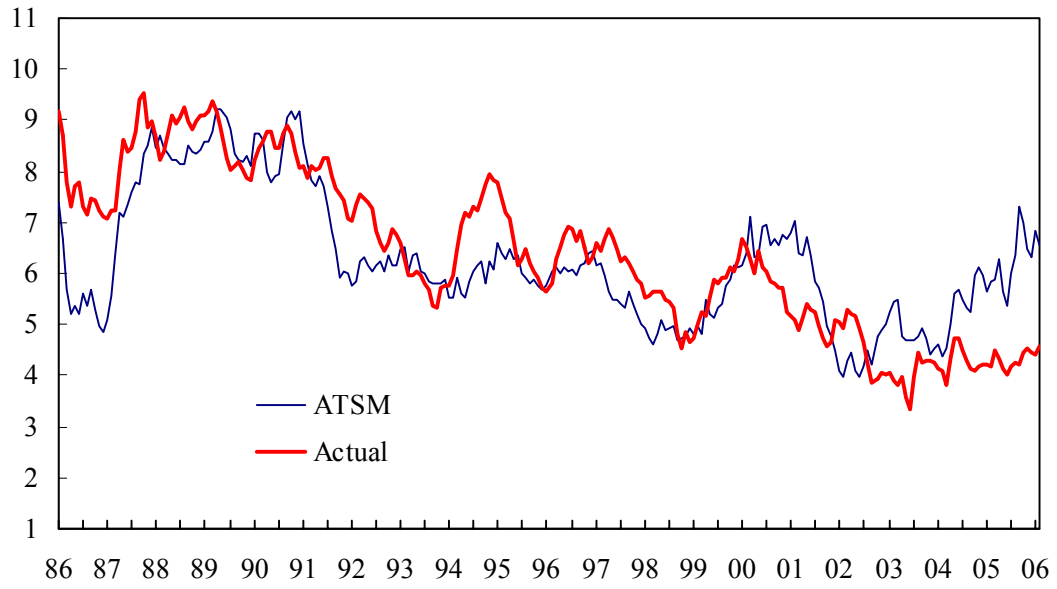


Figure 3: The ATSM predicted values

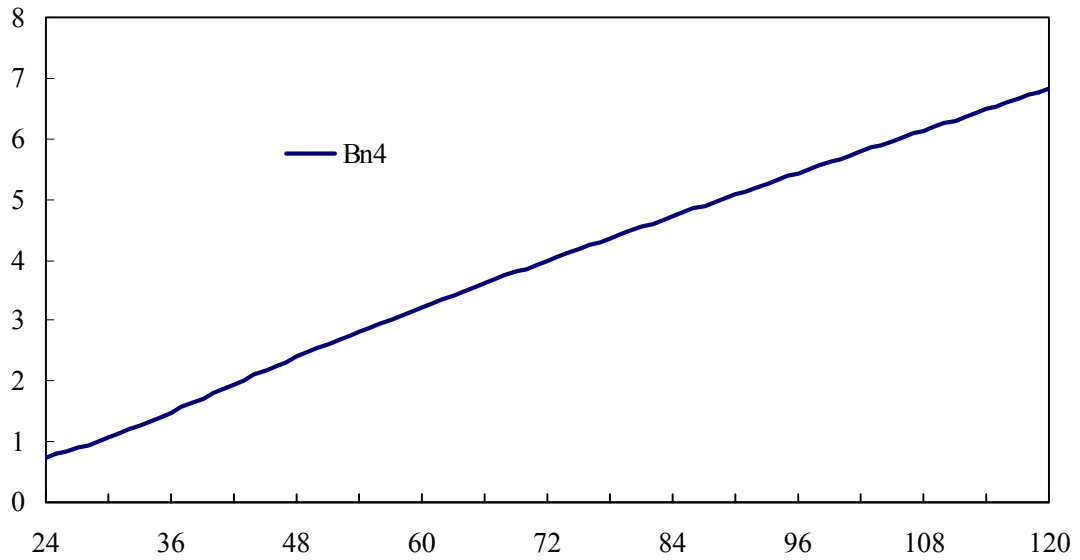


Figure 4: Evolutions of $\bar{B}_n(4)$

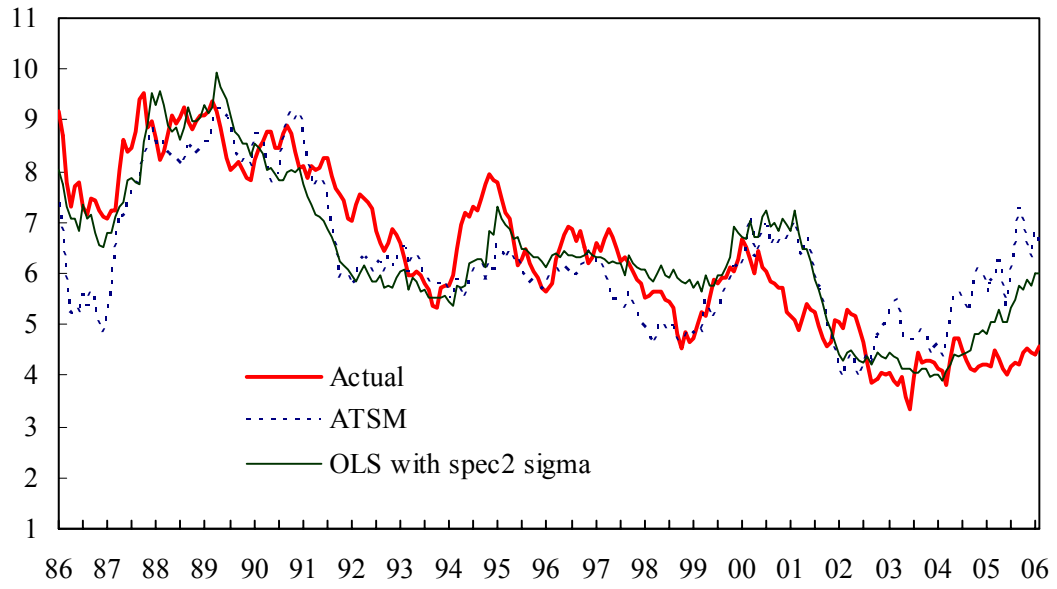


Figure 5: The ATSM vs OLS